Nonlinear Multimode Response of Clamped Rectangular Plates to Acoustic Loading

Chuh Mei*
Old Dominion University, Norfolk, Virginia
and
Donald B. Paul†
Air Force Wright Aeronautical Laboratories, Wright-Patterson Air Force Base, Ohio

Large-deflection and multiple modes are included in this analysis in order to improve the prediction of the random response of clamped rectangular panels subjected to broadband acoustic excitation. The von Kármán large-deflection plate equations, Galerkin's method, and the equivalent linearization technique are employed in the development. Mean-square deflections, mean-square strains, and equivalent linear frequencies are obtained for rectangular panels at various acoustic loadings.

Nomenclature

a,b	= plate length and width
$B_{pqmnk\ell}$	= integers, Eq. (8)
$[C]$	= generalized damping matrix
C_{pq}	= function, Eq. (31)
D	= plate flexural rigidity
E	= Young's modulus
$f_m(x), g_n(y)$	= displacement functions, Eqs. (5) and (6)
F	= stress function
F_{pq}	= stress function coefficients, Eq. (8)
h	= plate thickness
$I_x(W_{mn})$, $I_y(W_{mn})$	= functions, Eqs. (A3) and (A4)
$[K]$	= generalized stiffness matrix
L	= mathematical operator, Eq. (1)
$[M]$	= generalized mass matrix
p	= pressure
P	= pressure in normal coordinates
P_x, P_y	= average edge loads, Eq. (7)
q	= normal coordinates
S_f	= nondimensional pressure spectral density, Eq. (36)
t	= time
u,v	= in-plane displacements
w	= lateral deflection
W_{mn}	= generalized displacements
x,y	= coordinates
$Z_x(W_{mn})$, $Z_y(W_{mn})$	= functions, Eqs. (A5) and (A6)
α	= length-to-width ratio, a/b
β	= vector function, Eq. (12)
ϵ	= normal strain
ζ	= damping ratio, c/c_c
ν	= Poisson's ratio
ρ	= mass density
ϕ	= normal mode
ω	= linear frequency, rad/s
Ω	= equivalent linear or nonlinear frequency, rad/s

Subscripts

b	= bending component
EL	= equivalent linear
L	= linear
m	= membrane component

Presented as Paper 83-1033 at the AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, NV, May 2-4, 1983; received May 14, 1984; revision received May 28, 1985. This paper is declared a work of the U.S. Government and therefore is in the public domain.

*Associate Professor, Department of Mechanical Engineering and Mechanics. Member AIAA.

†Aerospace Engineer, Structural Integrity Branch, Flight Dynamics Laboratory. Member AIAA.

Introduction

ACOUSTICALLY induced fatigue failure in aircraft structures have been a design consideration for the past two decades. The problem was introduced with the advent of the jet engine, which produced high-intensity acoustic pressure fluctuations on aircraft surfaces. As the engine performance requirements increased, the intensity of the acoustic pressures increased. Airframe minimum weight requirements resulted in higher stresses in structural components. The number of acoustic fatigue failures have also resulted in unacceptable maintenance and inspection burdens associated with the operation of the aircraft. Therefore, accurate prediction methods are needed to determine the acoustic fatigue life of structures. Numerous analytical studies[1-8] and experimental investigations[2,8-16] on sonic fatigue design of aircraft structures have been undertaken during the past decade to help provide the needed reliability.

The majority of analytical studies to date have been formulated within the framework of linear or small-deflection structural theory. Test results on various aircraft panels reported in Refs. 2, 8-11, and 13-16, however, have shown that high noise levels in excess of 120 dB produce nonlinear large-deflection behavior in such panels. The linear analyses predict the root-mean-square (RMS) strains/stresses well above those of the experiment, and the frequencies of vibration well below those of the experiment.[2,8,13,15,16] It is known that the estimation of service life is based on RMS stress/strain and predominant response frequency in conjunction with the stress vs cycles to failure (S-N) data. Current analytical design methods[2,4,7,8] for sonic fatigue prevention are based essentially on linear theory. The use of linear analyses, therefore, would lead to poor prediction of panel fatigue life. To have an accurate determination of the random response of a structure, large-deflection or nonlinear structural theory should be employed.

Recently, analytical efforts[1,3,11] with a single-mode approach have demonstrated that the prediction of panel random response is greatly improved by including the large-deflection effect in the formulation. Test results[11] also showed that there is more than one mode responding. Multiple modes were also observed by White[17] in experimental studies on aluminum and carbon fiber-reinforced plates under acoustic loading. White also showed that the fundamental mode responded significantly and contributed more than one-half of the total mean-square strain response. Therefore, in order to have an accurate determination of the random response, multiple modes should also be used in the analysis.

Various techniques for predicting the response of multiple-degree-of-freedom (MDOF) systems of nonlinear second-order equations can be basically divided into analytical methods [Fokker-Planck-Kolmogorov (FPK) equation, equivalent or statistical linearization, and perturbation] and numerical simulation (or the Monte Carlo method). The most general extension of the FPK equation to nonlinear MDOF dynamic systems was developed by Caughey.[18,19] One advantage of this method over all other approaches is that it gives an exact solution. However, this should not be construed to mean that all problems relating to the response of nonlinear systems to random excitation have been solved. In fact, exact solutions of the steady-state probability function have been found only for certain restricted classes of problems provided: 1) The only energy dissipation in the system arises from damping forces that are proportional to the velocity; 2) the exciting forces are uncorrelated Gaussian white noise; 3) the spectral density matrix of the excitation is proportional to the damping matrix of the system; and, 4) the restoring force vector of the system is derivable from a potential. Problems of simple structures that satisfy these four conditions were solved by Herbert.[20,21] Yet many problems of practical interest do not satisfy those conditions necessary for a solution. Thus, a number of approximate techniques have been developed to treat a broader class of problems than is presently possible with the exact analysis. These are the equivalent linearization, perturbation, and numerical simulation methods.

The classical perturbation method for deterministic nonlinear problems was extended to random vibration problems by Crandell.[22] Lyon[23] used this approach to study the responses of a nonlinear string. Tung et al.[24] used the perturbation procedure to a two-DOF system. In principle, the perturbation approach can be extended to systems of coupled nonlinear equations in which the nonlinearities contain a small parameter ϵ. For complex structures, however, the algebraic operations may become so unwieldy that the prodigious amounts of labor make the method no longer practical.

Numerical, or the Monte Carlo, simulation[25] consists of generating a large number of sample excitations, computing the corresponding response samples, and then processing them to obtain the desired response statistics. The procedure is, in principle, very general. The major drawbacks of this method are computation time and cost. Spanos[25] has estimated that, for problems to which both equivalent linearization and numerical simulation can be applied, the computational efficiency of the equivalent linearization will be of the order of 100 to 1000 times better than the Monte Carlo method. Roberts,[26] To,[27] and Crandell and Zhu[28] have presented their comprehensive and excellent reviews on nonlinear random vibrations.

This paper presents an analytical solution for the large-amplitude random response of clamped rectangular plates considering multiple modes in the analysis. The von Kármán large-deflection plate equations are solved by a technique that reduces the fourth-order nonlinear partial differential equations to a set of second-order nonlinear differential equations with time as the independent variable. A Fourier-type series representation of the out-of-plane deflection and stress function is assumed. The compatibility equation is solved by direct substitution, and the equilibrium equation is solved through the use of the Bubnov-Galerkin approach. The acoustic excitation is assumed to be Gaussian. The Krylov-Bogoliubov-Caughey equivalent linearization method[25,29-31] is then used so that the derived set of second-order nonlinear differential equations are linearized to an equivalent set of second-order linear differential equations. Transformation of coordinates from the generalized displacements to the normal-mode coordinates and an iterative scheme are employed to obtain RMS maximum panel deflection, RMS maximum strain, and equivalent linear (or nonlinear) frequencies for rectangular plates at various excitation pressure spectral densities. Convergence of the results is demonstrated by using 4, 6, 10, and 15 terms in the transverse deflection function.

Mathematical Formulation and Solution Procedure

Assuming that the effects of both in-plane and rotary inertia forces can be neglected, the dynamic von Kármán equations of a rectangular isotropic plate are

$$L(w,F) = D\nabla^4 w + \rho h w,_{tt} + g w,_t - p(t) - h(F,_{yy} w,_{xx} + F,_{xx} w,_{yy} - 2F,_{xy} w,_{xy}) = 0 \quad (1)$$

$$\nabla^4 F = E(w,_{xy}^2 - w,_{xx} w,_{yy}) \quad (2)$$

The transverse deflection that satisfies the clamped boundary conditions

$$w = w,_x = 0 \text{ on } x=0 \text{ and } a \quad (3a)$$

$$w = w,_y = 0 \text{ on } y=0 \text{ and } b \quad (3b)$$

is assumed to be

$$w(x,y,t) = h \sum_m \sum_n W_{mn}(t) f_m(x) g_n(y) \qquad m,n = 1,2,3,\ldots \quad (4)$$

where

$$f_m(x) = \cos[(m-1)\pi x/a] - \cos[(m+1)\pi x/a] \quad (5)$$

$$g_n(y) = \cos[(n-1)\pi y/b] - \cos[(n+1)\pi y/b] \quad (6)$$

Upon examination of the foregoing expression for the transverse deflection, it is found that the compatibility equation (2) can be identically satisfied if the stress function F is taken in the following form:

$$F = -P_x \frac{y^2}{2} - P_y \frac{x^2}{2} + Eh^2 \sum_p \sum_q F_{pq} \cos\frac{p\pi x}{a} \cos\frac{q\pi y}{b} \qquad p,q = 0,1,2,\ldots \quad (7)$$

Direct substitution of Eqs. (4) and (7) into Eq. (2), performing the required differentiations, multiplications, and a Fourier analysis of the resulting terms, yields a quadratic relationship between F_{pq} and W_{mn}

$$F_{pq} = \frac{1}{(p^2/\alpha + q^2\alpha)^2} \sum_m \sum_n \sum_k \sum_\ell B_{pqmnk\ell} W_{mn} W_{k\ell} \quad (8)$$

in which $B_{pqmnk\ell}$ are integers (tabulated in Ref. 32) and $\alpha = a/b$. A complete description of the solution technique used to solve Eq. (2) is given in Ref. 32.

The average edge loads P_x and P_y in Eq. (7) are determined from in-plane boundary conditions. The particular in-plane boundary condition of most interest in the study of sonic fatigue of structural panels is the one in which the edges are restrained from movement, that is,

$$u=0 \text{ on } x=0 \text{ and } a \tag{9a}$$

$$v=0 \text{ on } y=0 \text{ and } b \tag{9b}$$

or

$$\int_0^a \frac{\partial u}{\partial x}\, dx = \int_0^a \left[\frac{1}{E}(F,_{yy} - \nu F,_{xx}) - \frac{1}{2}\, w^2,_x\right] dx = 0 \tag{10a}$$

$$\int_0^b \frac{\partial v}{\partial y}\, dy = \int_0^b \left[\frac{1}{E}(F,_{xx} - \nu F,_{yy}) - \frac{1}{2}\, w^2,_y\right] dy = 0 \tag{10b}$$

Performing the differentiation and integration as indicated in Eq. (10) yields relationships for P_x and P_y in terms of the generalized displacements W_{mn}. These relations are given in the Appendix.

With the assumed deflection w given by Eq. (4) and the stress function F given by Eq. (7), Eq. (1) is then satisfied by applying the Bubnov-Galerkin method:

$$\iint L(w,F) f_r g_s \, dx dy = 0 \quad r,s = 1,2,3,\ldots \tag{11}$$

The integration of each of the terms in Eq. (11) can be found in Ref. 32. A set of nonlinear time-differential equations is obtained after performing the integration over the total area of the panel, and it can be written in matrix notation as

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]_L\{W\} + \{\beta(W)\} = \{p(t)\} \tag{12}$$

where $[M]$, $[C]$, and $[K]_L$ are the generalized mass, damping, and linear stiffness matrices, respectively, and $\{\beta\}$ is a vector function that is cubic in the generalized displacements $\{W\}$.

If the acoustic pressure excitation $p(t)$ is stationary Gaussian, ergodic, and has a zero mean, then application of the Krylov-Bogoliubov-Caughey equivalent linearization technique yields an equivalent set of linear equations as

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + ([K]_L + [K]_{EL})\{W\} = \{p(t)\} \tag{13a}$$

or

$$[M]\{\ddot{W}\} + [C]\{\dot{W}\} + [K]\{W\} = \{p(t)\} \tag{13b}$$

The elements of the generalized equivalent linear or nonlinear stiffness matrix $[K]_{EL}$ can be derived from the expression[30]

$$(K_{EL})_{rsij} = \xi\left[\frac{\partial \beta_{rs}}{\partial W_{ij}}\right] \tag{14}$$

where $\xi[\;]$ is an expected value operator. The elements $(K_{EL})_{rsij}$ are too lengthy to reproduce here but may be found in Ref. 33. The approximate generalized displacements $[W]$, computed from the linearized Eq. (13), are also Gaussian and nearly stationary because the panel motion is stable.

To determine the mean-square generalized displacements $\overline{W^2_{mn}}$ in Eq. (13), an iterative process is introduced. The undamped linear equation of Eq. (13a) is solved first, which requires simply the determination of the eigenvalues and eigenvectors of the undamped linear equation

$$\omega_j^2 [M]\{\phi\}_j = [K]_L\{\phi\}_j \tag{15}$$

where ω_j is the linear frequency of vibration and $[\phi]_j$ is the normal mode shape.

Apply a coordinate transformation, from the generalized displacements to the normal coordinates (this analysis will use the first four modes), by

$$\underset{m\times 1}{\{W\}} = \underset{m\times 4}{[\phi]}\,\underset{4\times 1}{\{q\}} \quad 4 \le m \tag{16}$$

where each column of $[\phi]$ is a normal mode $\{\phi\}_j$. The damped linear equation of Eq. (13a) becomes

$$[\;M\;]\{\ddot{q}\} + [\;C\;]\{\dot{q}\} + [\;K\;]_L\{q\} = \{P(t)\} \tag{17}$$

where

$$[\;M\;] = [\phi]^T[M][\phi] \tag{18a}$$

$$[\;C\;] = [\phi]^T[C][\phi] = 2[\;\zeta\omega\;][\;M\;] \tag{18b}$$

$$[\;K\;]_L = [\phi]^T[K]_L[\phi] = [\;\omega^2\;][\;M\;] \tag{18c}$$

$$\{P\} = [\phi]^T\{p\} \tag{18d}$$

The jth row of Eq. (17) is

$$\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j = P_j/M_j \tag{19}$$

The mean-square normal coordinate is simply

$$\overline{q_j^2} \cong \frac{\pi S_p(\omega_j)}{4M_j^2\zeta_j\omega_j^3} \tag{20}$$

where $S_p(\omega)$ is the spectral density function of the excitation $P_j(t)$. The covariance matrix of the linear generalized displacements is

$$\overline{[W_{mn}W_{k\ell}]_L} = \frac{\pi}{4}\sum_j \{\phi\}_j \frac{S_p(\omega_j)}{M_j^2\zeta_j\omega_j^3}[\phi]_j^T \tag{21}$$

This initial estimate of expected value on generalized displacements can now be used to compute the generalized equivalent linear stiffness matrix $[K]_{EL}$ through Eq. (14). The undamped linearized equation of Eq. (13) is solved again

$$\Omega_j^2[M]\{\phi\}_j = ([K]_L + [K]_{EL})\{\phi\}_j \tag{22}$$

where Ω_j is the equivalent linear or nonlinear frequency of vibration and $\{\phi\}_j$ is the associated equivalent linear normal mode shape. Then Eq. (13) is transformed again and has the form

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{P(t)\} \tag{23}$$

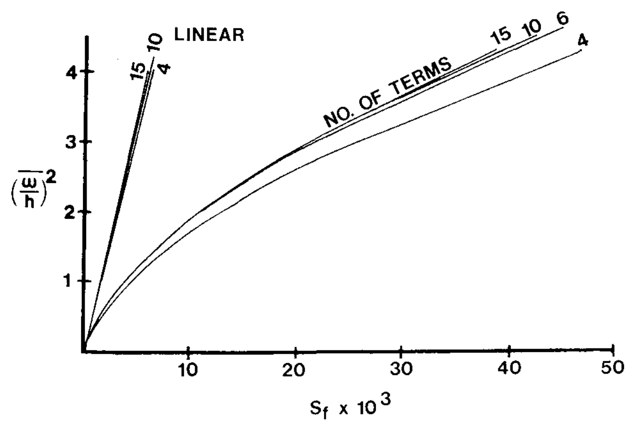

Fig. 1 Convergence of the mean-square deflection for the square plate ($\zeta = 0.009$).

in which

$$[\,K\,] = [\phi]^T([K]_L + [K]_{EL})[\phi] = [\,\Omega^2\,][\,M\,] \tag{24}$$

The jth row of Eq. (23) is

$$\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \Omega_j^2 q_j = P_j/M_j \tag{25}$$

and the displacement covariance matrix is

$$\overline{[W_{mn}W_{k\ell}]} = \frac{\pi}{4}\sum_j \{\phi\}_j \frac{S_p(\Omega_j)}{M_j^2\zeta_j\omega_j\Omega_j^2}\{\phi\}_j^T \tag{26}$$

The diagonal terms of $\overline{[W_{mn}W_{k\ell}]}$ are the mean-square generalized displacements $\overline{W_{mn}^2}$. As the iterative process converges on the iterth cycle, the relations

$$(\overline{q_j^2})_{\text{iter}} \cong (\overline{q_j^2})_{\text{iter}-1} \tag{27a}$$

$$(\Omega_j^2)_{\text{iter}} \cong (\Omega_j^2)_{\text{iter}-1} \tag{27b}$$

$$(\overline{W_{mn}^2})_{\text{iter}} \cong (\overline{W_{mn}^2})_{\text{iter}-1} \tag{27c}$$

become satisfied. Convergence is considered achieved when the difference of the RMS displacements satisfies the requirement

$$\frac{(\text{RMS}\,W_{mn})_{\text{iter}} - (\text{RMS}\,W_{mn})_{\text{iter}-1}}{(\text{RMS}\,W_{mn})_{\text{iter}}} \le 10^{-3} \quad \text{for all } m,n \tag{28}$$

Once the RMS generalized displacements are determined, the RMS deflection and the maximum RMS strain of the plate can be determined from Eqs. (4) and (7) and the strain-displacement relations as follows.

For a rectangular plate ($a \ge b$) with clamped support along all four edges, the maximum bending strain occurs at the extreme-fiber perpendicular to its long edge at the midpoint.

$$(\epsilon_y)_b = \pm\frac{h}{2}\frac{\partial^2 w}{\partial y^2} \tag{29a}$$

or

$$\left(\frac{\epsilon_y b^2}{h^2}\right)_b = \pm\frac{\pi^2}{2}\sum_m\sum_n f_m(x)\left[(n-1)^2\cos\frac{(n-1)\pi y}{b} - (n+1)^2\cos\frac{(n+1)\pi y}{b}\right]W_{mn} \tag{29b}$$

The membrane strain is given by

$$(\epsilon_y)_m = (1/E)(F_{,xx} - \nu F_{,yy}) \tag{30a}$$

or

$$\left(\frac{\epsilon_y b^2}{h^2}\right)_m = \frac{\pi^2}{8}\sum_m\sum_n W_{mn}Z_y(W_{mn}) + \sum_m\sum_n\sum_k\sum_\ell\sum_p\sum_q C_{pq}B_{pqmnk\ell}W_{mn}W_{k\ell} \tag{30b}$$

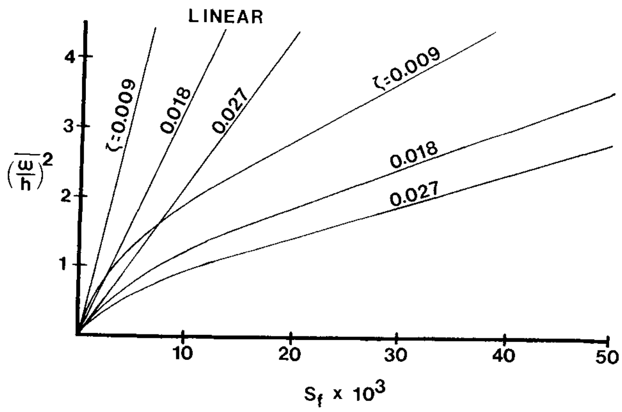

Fig. 3 Mean-square center deflection vs pressure spectral density for a clamped square plate.

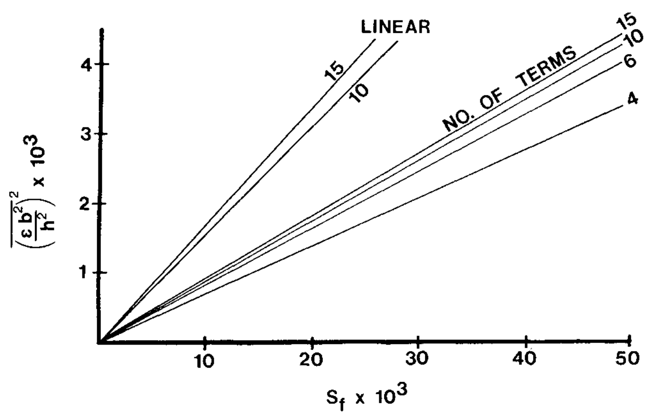

Fig. 2 Convergence of the maximum mean-square strain for the square plate ($\zeta = 0.009$).

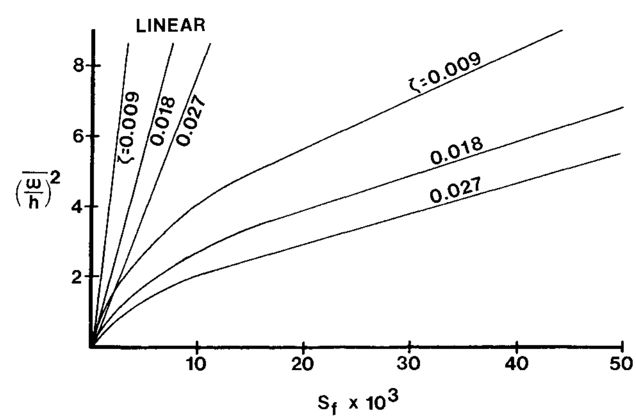

Fig. 4 Mean-square center deflection vs pressure spectral density for a clamped rectangular ($\alpha = 2$) plate.

Table 1 Generalized displacements used in convergence studies

Number of terms	Generalized displacements														
	W_{11}	W_{13}	W_{31}	W_{33}	W_{15}	W_{51}	W_{35}	W_{53}	W_{17}	W_{71}	W_{55}	W_{37}	W_{73}	W_{19}	W_{91}
4	X	X	X	X											
6	X	X	X	X	X	X									
10	X	X	X	X	X	X	X	X	X	X					
15	X	X	X	X	X	X	X	X	X	X	X	X	X	X	

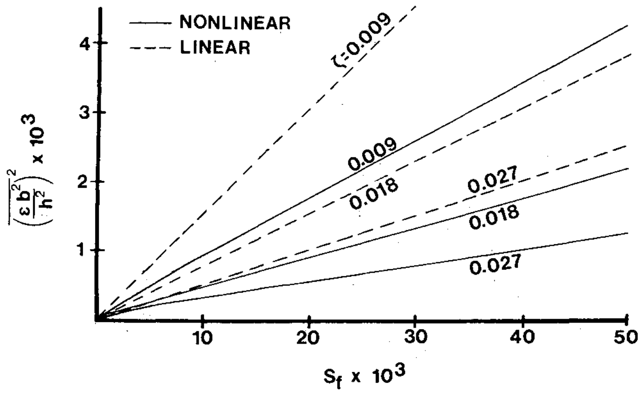

Fig. 5 Maximum mean-square strain vs pressure spectral density for a clamped square plate.

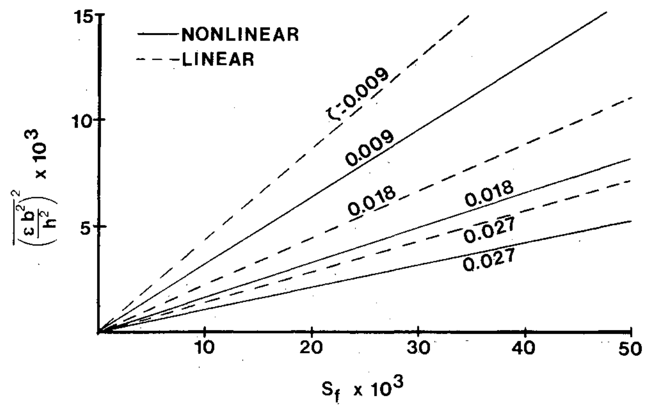

Fig. 6 Maximum mean-square strain vs pressure spectral density for a clamped rectangular ($\alpha=2$) plate.

in which

$$C_{pq} = \frac{\pi^2(\nu q^2 - p^2/\alpha^2)}{(p^2/\alpha + q^2\alpha)^2} \cos\frac{p\pi x}{a} \cos\frac{q\pi y}{b} \tag{31}$$

From

$$\epsilon_y = (\epsilon_y)_b + (\epsilon_y)_m \tag{32}$$

the maximum mean-square strain becomes

$$\left(\frac{\overline{\epsilon_y b^2}}{h^2}\right)^2 = \left(\frac{\overline{\epsilon_y b^2}}{h^2}\right)^2_b + 2\xi\left[\left(\frac{\epsilon_y b^2}{h^2}\right)_b\left(\frac{\epsilon_y b^2}{h^2}\right)_m\right] + \left(\frac{\overline{\epsilon_y b^2}}{h^2}\right)^2_m \tag{33}$$

For Gaussian random processes with zero mean, we have

$$\xi(W_{ij}W_{k\ell}W_{mn}) = 0 \tag{34}$$

$$\xi(W_{ij}W_{k\ell}W_{mn}W_{rs}) = \xi(W_{ij}W_{k\ell})\xi(W_{mn}W_{rs}) + \xi(W_{ij}W_{mn})\xi(W_{k\ell}W_{rs}) + \xi(W_{ij}W_{rs})\xi(W_{k\ell}W_{mn}) \tag{35}$$

and the maximum RMS strain can be determined from Eq. (33).

Results and Discussion

Using the present formulation, the nonlinear response of square and rectangular ($\alpha=2$) plates with all edges clamped and subjected to broadband random excitation are studied. In the results presented, the white-noise excitation is band-limited with a frequency bandwidth of 25 Hz to 6000 Hz, the damping ratio is assumed to be constant for all four normal modes, and Poisson's ratio is equal to 0.3. Mean-square center deflection and maximum mean-square strain are presented in a nondimensional form. The nondimensional forcing spectral density parameter is defined as

$$S_f = \frac{2\pi S_p(\omega)}{\rho^2 h^4 (D/\rho h b^4)^{3/2}} \tag{36}$$

Also, since the loading is symmetric, only symmetric generalized displacements W_{mn} are retained in the transverse deflection function.

The convergence of the solution technique was examined in order to determine the degree of accuracy possible with a highly truncated transverse deflection function series. The mean-square center deflection vs the nondimensional spectral density parameter using 4, 6, 10, and 15 terms in the deflection function for a square plate is shown in Fig. 1. The particular generalized displacements that make up the various orders of the deflection function are shown in Table 1. Figure 1 clearly indicates that a six-term solution gives accurate results for the nonlinear maximum deflection while a four-term solution will provide accurate linear results. The maximum strain occurs at the extreme fiber of the panel and at the midpoint of the long edge. The direction is perpendicular to the edge. Figure 2 shows the maximum mean-square strain vs S_f for the square plate using 4, 6, 10, and 15 terms in the deflection function. The convergence of the mean-square strain is much slower as compared with that of the mean-square deflection.

Figures 3 and 4 show the maximum mean-square nondimensional deflection vs the nondimensional spectral density of acoustic pressure excitation for rectangular panels of aspect ratios of 1 and 2 with the damping ratio equal to 0.009, 0.018, and 0.027. Figures 5 and 6 show the maximum nondimensional mean-square strain vs the nondimensional spectral density for rectangular panels of aspect ratios of 1 and 2 with the damping ratio equal to 0.009, 0.018, and 0.027. Ten terms were included in the deflection function to generate the results shown in Figs. 3 and 6.

Concluding Remarks

An analytical solution technique is presented for determining the large-amplitude random response of clamped rectangular panels while including multiple modes in the analysis. Accurate mean-square deflections can be obtained with the use of six terms in the deflection function, while it is necessary to consider as many as ten or more terms for the accurate determination of the strains. In the numerical examples presented, a constant damping ratio for all four modes has been used. However, nonlinear damping phenomena have been observed in experiments.[11,17] Therefore, further effort is needed to better understand the effects of nonlinear damping on panel response.

Appendix

Average edge loads P_x and P_y in terms of the generalized displacements W_{mn}:

$$P_x = -\frac{E}{1-\nu^2}\left(\frac{\nu}{b} I_y + \frac{1}{a} I_x\right) \tag{A1}$$

$$P_y = -\frac{E}{1-\nu^2}\left(\frac{\nu}{a} I_x + \frac{1}{b} I_y\right) \tag{A2}$$

in which

$$I_x(W_{mn}) = \frac{h^2\pi^2}{8a} \sum_m \sum_n W_{mn} Z_x(W_{mn}) \tag{A3}$$

$$I_y(W_{mn}) = \frac{h^2\pi^2}{8b}\sum_m\sum_n W_{mn} Z_y(W_{mn}) \tag{A4}$$

$$\begin{aligned} Z_x(W_{mn}) &= [(m+1)^2+(m-1)^2]W_{m,2-n} \\ &-(m+1)^2W_{m+2,2-n}-(m-1)^2W_{m-2,2-n} \\ &+2[(m+1)^2+(m-1)^2]W_{mn}-2(m+1)^2W_{m+2,n} \\ &-2(m-1)^2W_{m-2,n}-[(m+1)^2+(m-1)^2]W_{m,n-2} \\ &+(m+1)^2W_{m+2,n-2}+(m-1)^2W_{m-2,n-2} \\ &-[(m+1)^2+(m-1)^2]W_{m,n+2} \\ &+(m+1)^2W_{m+2,n+2}+(m-1)^2W_{m-2,n+2} \end{aligned} \tag{A5}$$

$$\begin{aligned} Z_y(W_{mn}) &= [(n+1)^2+(n-1)^2]W_{2-m,n} \\ &-(n+1)^2W_{2-m,n+2}-(n-1)^2W_{2-m,n-2} \\ &+2[(n+1)^2+(n-1)^2]W_{mn}-2(n+1)^2W_{m,n+2} \\ &-2(n-1)^2W_{m,n-2}-[(n+1)^2+(n-1)^2]W_{m-2,n} \\ &+(n+1)^2W_{m-2,n+2}+(n-1)^2W_{m-2,n-2} \\ &-[(n+1)^2+(n-1)^2]W_{m+2,n} \\ &+(n+1)^2W_{m+2,n+2}+(n-1)^2W_{m+2,n-2} \end{aligned} \tag{A6}$$

where $W_{mn}=0$ for m or $n<1$.

Acknowledgments

This work was sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, under Grant AFOSR-80-0107.

References

[1]Wentz, K. R., Paul, D. B., and Mei, C., "Large Deflection Random Response of Symmetric Laminated Composite Plates," *Shock and Vibration Bulletin*, Bulletin 52, May 1982, pp. 99-111.

[2]Holehouse, I., "Sonic Fatigue Design Techniques for Advanced Composite Aircraft Structures," Wright-Patterson AFB, OH, AFWAL-TR-80-3019, April 1980.

[3]Mei, C., "Response of Nonlinear Structural Panels Subjected to High Intensity Noise," Wright-Patterson AFB, OH, AFWAL-TR-80-3018, March 1980.

[4]Rudder, F. F. Jr. and Plumblee, H. E. Jr., "Sonic Fatigue Design Guide for Military Aircraft," Wright-Patterson AFB, OH, AFFDL-TR-74-112, May 1975.

[5]Volmir, A. S., "The Nonlinear Dynamics of Plates and Shells," Foreign Technology Div., Wright-Patterson AFB, OH, AD-781338, April 1974, Chap. X.

[6]Fox, H. L., Smith, P. W. Jr., Pyle, R. W., and Nayak, P. R., "Contributions to the Theory of Randomly Forced, Nonlinear, Multiple-Degree-of-Freedom Coupled Mechanical Systems," Wright-Patterson AFB, OH, AFFDL-TR-72-45, Aug. 1973.

[7]Thomson, A. G. R. and Lambert, R. F., "Acoustic Fatigue Design Data," NATO Advisory Group for Aeronautics Research and Development, AGARD-AG-162, Pts. I and II, 1972.

[8]Jacobs, L. D. and Lagerquist, D. R., "Finite Element Analysis of Complex Panel to Random Loads," Wright-Patterson AFB, OH, AFFDL-TR-68-44, Oct. 1968.

[9]Soovere, J., "The Effect of Acoustic-Thermal Environments on Advanced Composite Fuselage Panels," *Proceedings of the AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference*, Lake Tahoe, NV, May 1983, pp. 466-472.

[10]Soovere, J., "Sonic Fatigue Testing of an Advanced Composite Aileron," *Journal of Aircraft*, Vol. 19, April 1982, pp. 304-310.

[11]Mei, C. and Wentz, K. R., "Analytical and Experimental Nonlinear Response of Rectangular Panels to Acoustic Excitation," *Proceedings of the AIAA/ASME/ASCE/AHS 23rd Structures, Structural Dynamics and Materials Conference*, New Orleans, LA, May 1982, pp. 514-520.

[12]Wentz, K. R. and Wolfe, H. F., "Development of Random Fatigue Data for Adhesively Bonded and Weldbonded Structures Subjected to Dynamic Excitation," *ASME Journal of Engineering Materials and Technology*, Vol. 100, Jan. 1978, pp. 70-76.

[13]Jacobson, M. J., "Sonic Fatigue Design Data for Bonded Aluminum Aircraft Structures," Wright-Patterson AFB, OH, AFFDL-TR-77-45, June 1977.

[14]Van der Heyde, R. C. W. and Wolf, N. D., "Comparison of the Sonic Fatigue Characteristics of Four Structural Designs," Wright-Patterson AFB, OH, AFFDL-TR-76-66, Sept. 1976.

[15]Van der Heyde, R. C. W. and Smith, D. L., "Sonic Fatigue Resistance of Skin-Stringer Panels," Wright-Patterson AFB, OH, AFFDL-TM-73-149-FYA, April 1974.

[16]Jacobson, M. J., "Advanced Composite Joints; Design and Acoustic Fatigue Characteristics," Wright-Patterson AFB, OH, AFFDL-TR-71-126, April 1972.

[17]White, R. G., "Comparison of the Statistical Properties of the Aluminum Alloy and CFRP Plates to Acoustic Excitation," *Journal of Composites*, Oct. 1978, pp. 251-258.

[18]Caughey, T. K., "Nonlinear Theory of Random Vibrations," *Advances in Applied Mechanics*, Vol. 11, Academic Press, New York, 1971, pp. 209-253.

[19]Caughey, T. K., "Derivation and Application of the Fokker-Planck Equation to Discrete Nonlinear Dynamic Systems Subjected to White Random Excitation," *Journal of the Acoustical Society of America*, Vol. 35, Nov. 1963, pp. 1683-1692.

[20]Herbert, R. E., "Random Vibrations of Plates with Large Amplitude," *Journal of Applied Mechanics*, Vol. 32, Sept. 1965, pp. 547-552.

[21]Herbert, R. E., "Random Vibrations of a Nonlinear Elastic Beam," *Journal of the Acoustical Society of America*, Vol. 36, Nov. 1964, pp. 2090-2094.

[22]Crandall, S. H., "Perturbation Techniques for Random Vibration of Nonlinear Systems," *Journal of the Acoustical Society of America*, Vol. 35, Nov. 1963, pp. 1700-1705.

[23]Lyon, R. H., "Response of a Nonlinear String to Random Excitation," *Journal of the Acoustical Society of America*, Vol. 32, Aug. 1960, pp. 953-960.

[24]Tung, C. C., Penzien, J., and Horonjeff, R., "The Effect of Runway Unevenness on the Dynamic Response of Supersonic Jet Transport," NASA CR-119, University of California, Berkeley, 1964.

[25]Spanos, P. T. D., "Stochastic Linearization in Structural Dynamics," *Applied Mechanics Review*, Vol. 34, 1981, pp. 1-8.

[26]Roberts, J. B., "Techniques for Nonlinear Random Vibration Problems," *Shock and Vibration Digest*, Vol. 16, Sept. 1984, pp. 3-14.

[27]To, C. W. S., "The Response of Nonlinear Structures to Random Excitation," *Shock and Vibration Digest*, Vol. 16, April 1984, pp. 13-33.

[28]Crandall, S. H. and Zhu, W. Q., "Random Vibration: A Survey of Recent Developments," *Journal of Applied Mechanics*, Vol. 50, Dec. 1983, pp. 953-962.

[29]Iwan, W. D. and Spanos, P. T. D., "On the Existence of Uniqueness of Solutions Generated by Equivalent Linearization," *International Journal of Nonlinear Mechanics*, Vol. 13, No. 2, 1978, pp. 71-78.

[30]Atalik, T. S. and Utku, S., "Stochastic Linearization of Multi-Degree-of-Freedom Nonlinear Systems," *Earthquake Engineering and Structural Dynamics*, Vol. 4, 1976, pp. 411-420.

[31]Caughey, T. K., "Equivalent Linearization Techniques," *Journal of the Acoustical Society of America*, Vol. 35, Nov. 1963, pp. 1706-1711.

[32]Paul, D. B., "Large Deflections of Clamped Rectangular Plates with Arbitrary Temperature Distributions," Ph.D. Dissertation, Ohio State University, Columbus, 1980.

[33]Mei, C., "Large Deflection Multimode Response of Clamped Rectangular Panels to Acoustic Excitation," Wright-Patterson AFB, OH, AFWAL-TR-83-3121, Vol. I, Dec. 1983.